

## E01BEF – NAG Fortran Library Routine Document

**Note.** Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

### 1 Purpose

E01BEF computes a monotonicity-preserving piecewise cubic Hermite interpolant to a set of data points.

### 2 Specification

```
SUBROUTINE E01BEF(N, X, F, D, IFAIL)
INTEGER           N, IFAIL
real             X(N), F(N), D(N)
```

### 3 Description

This routine estimates first derivatives at the set of data points  $(x_r, f_r)$ , for  $r = 1, 2, \dots, n$ , which determine a piecewise cubic Hermite interpolant to the data, that preserves monotonicity over ranges where the data points are monotonic. If the data points are only piecewise monotonic, the interpolant will have an extremum at each point where monotonicity switches direction. The estimates of the derivatives are computed by a formula due to Brodlie, which is described in Fritsch and Butland [1], with suitable changes at the boundary points.

The routine is derived from routine PCHIM in Fritsch [2].

Values of the computed interpolant, and of its first derivative and definite integral, can subsequently be computed by calling E01BFF, E01BGF and E01BHF, as described in Section 8.

### 4 References

- [1] Fritsch F N and Butland J (1984) A Method for constructing local monotone piecewise cubic interpolants *SIAM J. Sci. Statist. Comput.* **5** 300–304
- [2] Fritsch F N (1982) PCHIP final specifications *Report UCID-30194* Lawrence Livermore National Laboratory

### 5 Parameters

- |  |               |
|--|---------------|
| 1: N — INTEGER   | <i>Input</i>  |
| <i>On entry:</i> $n$ , the number of data points.  |               |
| <i>Constraint:</i> $N \geq 2$ .  |               |
| 2: X(N) — <b>real</b> array  | <i>Input</i>  |
| <i>On entry:</i> $X(r)$ must be set to $x_r$ , the $r$ th value of the independent variable (abscissa), for $r = 1, 2, \dots, n$ . |               |
| <i>Constraint:</i> $X(r) < X(r + 1)$ .   |               |
| 3: F(N) — <b>real</b> array  | <i>Input</i>  |
| <i>On entry:</i> $F(r)$ must be set to $f_r$ , the $r$ th value of the dependent variable (ordinate), for $r = 1, 2, \dots, n$ .   |               |
| 4: D(N) — <b>real</b> array  | <i>Output</i> |
| <i>On exit:</i> estimates of derivatives at the data points. $D(r)$ contains the derivative at $X(r)$ .                            |               |

**5: IFAIL — INTEGER***Input/Output*

*On entry:* IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

*On exit:* IFAIL = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors detected by the routine:

IFAIL = 1

On entry,  $N < 2$ .

IFAIL = 2

The values of  $X(r)$ , for  $r = 1, 2, \dots, N$ , are not in strictly increasing order.

## 7 Accuracy

The computational errors in the array D should be negligible in most practical situations.

## 8 Further Comments

The time taken by the routine is approximately proportional to  $n$ .

The values of the computed interpolant at the points  $PX(i)$ , for  $i = 1, 2, \dots, M$ , may be obtained in the *real* array PF, of length at least M, by the call:

```
CALL E01BFF(N,X,F,D,M,PX,PF,IFAIL)
```

where N, X and F are the input parameters to E01BEF and D is the output parameter from E01BEF.

The values of the computed interpolant at the points  $PX(i)$ , for  $i = 1, 2, \dots, M$ , together with its first derivatives, may be obtained in the *real* arrays PF and PD, both of length at least M, by the call:

```
CALL E01BGF(N,X,F,D,M,PX,PF,PD,IFAIL)
```

where N, X, F and D are as described above.

The value of the definite integral of the interpolant over the interval A to B can be obtained in the *real* variable PINT by the call:

```
CALL E01BHF(N,X,F,D,A,B,PINT,IFAIL)
```

where N, X, F and D are as described above.

## 9 Example

This example program reads in a set of data points, calls E01BEF to compute a piecewise monotonic interpolant, and then calls E01BFF to evaluate the interpolant at equally spaced points.

## 9.1 Program Text

**Note.** The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```

*      E01BEF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
    INTEGER         NIN, NOUT
    PARAMETER       (NIN=5,NOUT=6)
    INTEGER         MMAX, NMAX
    PARAMETER       (MMAX=50,NMAX=50)
*      .. Local Scalars ..
    real            STEP
    INTEGER         I, IFAIL, M, N, R
*      .. Local Arrays ..
    real            D(NMAX), F(NMAX), PF(MMAX), PX(MMAX), X(NMAX)
*      .. External Subroutines ..
    EXTERNAL        E01BEF, E01BFF
*      .. Intrinsic Functions ..
    INTRINSIC       MIN
*      .. Executable Statements ..
    WRITE (NOUT,*) 'E01BEF Example Program Results'
*      Skip heading in data file
    READ (NIN,*)
    READ (NIN,*) N
    IF (N.GT.0 .AND. N.LE.NMAX) THEN
        DO 20 R = 1, N
            READ (NIN,*) X(R), F(R)
20    CONTINUE
    IFAIL = 0
*
*      CALL E01BEF(N,X,F,D,IFAIL)
*
*      READ (NIN,*) M
    IF (M.GT.0 .AND. M.LE.MMAX) THEN
        * Compute M equally spaced points from X(1) to X(N).
        STEP = (X(N)-X(1))/(M-1)
        DO 40 I = 1, M
            PX(I) = MIN(X(1)+(I-1)*STEP,X(N))
40    CONTINUE
    IFAIL = 0
*
*      CALL E01BFF(N,X,F,D,M,PX,PF,IFAIL)
*
*      WRITE (NOUT,*)                               Interpolated'
*      WRITE (NOUT,*) ,      Abscissa           Value'
*      DO 60 I = 1, M
*          WRITE (NOUT,99999) PX(I), PF(I)
60    CONTINUE
    END IF
    END IF
    STOP
*
99999 FORMAT (1X,F13.4,2X,F13.4)
END

```

## 9.2 Program Data

```
E01BEF Example Program Data
      9          N, the number of data points
    7.99  0.00000E+0 X(R), F(R), independent and dependent variable
    8.09  0.27643E-4
    8.19  0.43750E-1
    8.70  0.16918E+0
    9.20  0.46943E+0
   10.00  0.94374E+0
   12.00  0.99864E+0
   15.00  0.99992E+0
   20.00  0.99999E+0 End of data points
      11          M, the number of evaluation points
```

## 9.3 Program Results

E01BEF Example Program Results

Interpolated	
Abscissa	Value
7.9900	0.0000
9.1910	0.4640
10.3920	0.9645
11.5930	0.9965
12.7940	0.9992
13.9950	0.9998
15.1960	0.9999
16.3970	1.0000
17.5980	1.0000
18.7990	1.0000
20.0000	1.0000

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