

F07FGF (SPOCON/DPOCON) – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

F07FGF (SPOCON/DPOCON) estimates the condition number of a real symmetric positive-definite matrix A , where A has been factorized by F07FDF (SPOTRF/DPOTRF).

2 Specification

```
SUBROUTINE F07FGF(UPLO, N, A, LDA, ANORM, RCOND, WORK, IWORK, INFO)
ENTRY      spocon(UPLO, N, A, LDA, ANORM, RCOND, WORK, IWORK, INFO)
INTEGER    N, LDA, IWORK(*), INFO
real      A(LDA,*), ANORM, RCOND, WORK(*)
CHARACTER*1 UPLO
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine estimates the condition number (in the 1-norm) of a real symmetric positive-definite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The routine should be preceded by a call to F06RCF to compute $\|A\|_1$ and a call to F07FDF (SPOTRF/DPOTRF) to compute the Cholesky factorization of A . The routine then uses Higham's implementation of Hager's method [1] to estimate $\|A^{-1}\|_1$.

4 References

- [1] Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

- 1:** UPLO — CHARACTER*1 *Input*
On entry: indicates whether A has been factorized as $U^T U$ or LL^T as follows:
 if UPLO = 'U', then $A = U^T U$, where U is upper triangular;
 if UPLO = 'L', then $A = LL^T$, where L is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 2:** N — INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 3:** A(LDA,*) — *real* array *Input*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the Cholesky factor of A , as returned by F07FDF (SPOTRF/DPOTRF).

- 4:** LDA — INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F07FGF (SPOCON/DPOCON) is called.
Constraint: LDA \geq max(1,N).
- 5:** ANORM — *real* *Input*
On entry: the 1-norm of the **original** matrix A, which may be computed by calling F06RCF. ANORM must be computed either **before** calling F07FDF (SPOTRF/DPOTRF) or else from a copy of the original matrix A.
Constraint: ANORM \geq 0.0.
- 6:** RCOND — *real* *Output*
On exit: an estimate of the reciprocal of the condition number of A. RCOND is set to zero if exact singularity is detected or the estimate underflows. If RCOND is less than **machine precision**, then A is singular to working precision.
- 7:** WORK(*) — *real* array *Workspace*
Note: the dimension of the array WORK must be at least max(1,3*N).
- 8:** IWORK(*) — INTEGER array *Workspace*
Note: the dimension of the array IWORK must be at least max(1,N).
- 9:** INFO — INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to this routine involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ floating-point operations but takes considerably longer than a call to F07FEF (SPOTRS/DPOTRS) with 1 right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this routine is F07FUF (CPOCON/ZPOCON).

9 Example

To estimate the condition number in the 1-norm (or infinity-norm) of the matrix A, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

Here A is symmetric positive-definite and must first be factorized by F07FDF (SPOTRF/DPOTRF). The true condition number in the 1-norm is 97.32.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```

*      F07FGF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
INTEGER          NIN, NOUT
PARAMETER       (NIN=5,NOUT=6)
INTEGER          NMAX, LDA
PARAMETER       (NMAX=8,LDA=NMAX)
*      .. Local Scalars ..
real           ANORM, RCOND
INTEGER          I, INFO, J, N
CHARACTER        UPLO
*      .. Local Arrays ..
real           A(LDA,NMAX), WORK(3*NMAX)
INTEGER          IWORK(NMAX)
*      .. External Functions ..
real           F06RCF, X02AJF
EXTERNAL         F06RCF, X02AJF
*      .. External Subroutines ..
EXTERNAL         spocon, spotrf
*      .. Executable Statements ..
WRITE (NOUT,*) 'F07FGF Example Program Results'
Skip heading in data file
READ (NIN,*)
READ (NIN,*) N
IF (N.LE.NMAX) THEN
*
*      Read A from data file
*
      READ (NIN,*) UPLO
      IF (UPLO.EQ.'U') THEN
          READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
      ELSE IF (UPLO.EQ.'L') THEN
          READ (NIN,*) ((A(I,J),J=1,I),I=1,N)
      END IF
*
*      Compute norm of A
*
      ANORM = F06RCF('1-norm',UPLO,N,A,LDA,WORK)
*
*      Factorize A
*
      CALL spotrf(UPLO,N,A,LDA,INFO)
*
      WRITE (NOUT,*)
      IF (INFO.EQ.0) THEN
*
*      Estimate condition number
*
          CALL spocon(UPLO,N,A,LDA,ANORM,RCOND,WORK,IWORK,INFO)
*
          IF (RCOND.GE.X02AJF()) THEN
              WRITE (NOUT,99999) 'Estimate of condition number =',
+              1.0e0/RCOND
          ELSE

```

```
        WRITE (NOUT,*) 'A is singular to working precision'
      END IF
    ELSE
      WRITE (NOUT,*) 'A is not positive-definite'
    END IF
  END IF
  STOP
*
99999 FORMAT (1X,A,1P,e10.2)
END
```

9.2 Program Data

F07FGF Example Program Data

```
  4                               :Value of N
  'L'                             :Value of UPL0
  4.16
 -3.12  5.03
  0.56 -0.83  0.76
 -0.10  1.18  0.34  1.18  :End of matrix A
```

9.3 Program Results

F07FGF Example Program Results

Estimate of condition number = 9.73E+01
