

F07NSF (CSYTRS/ZSYTRS) – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

F07NSF (CSYTRS/ZSYTRS) solves a complex symmetric system of linear equations with multiple right-hand sides, $AX = B$, where A has been factorized by F07NRF (CSYTRF/ZSYTRF).

2 Specification

```
SUBROUTINE F07NSF(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, INFO)
ENTRY      csytrs(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, INFO)
INTEGER      N, NRHS, LDA, IPIV(*), LDB, INFO
complex      A(LDA,*), B(LDB,*)
CHARACTER*1   UPLO
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

To solve a complex symmetric system of linear equations $AX = B$, this routine must be preceded by a call to F07NRF (CSYTRF/ZSYTRF) which computes the Bunch–Kaufman factorization of A .

If $\text{UPLO} = \text{'U}'$, $A = PUDU^T P^T$, where P is a permutation matrix, U is an upper triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^T P^T X = Y$.

If $\text{UPLO} = \text{'L}'$, $A = PLDL^T P^T$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^T P^T X = Y$.

4 References

- [1] Golub G H and van Loan C F (1996) *Matrix Computations* Johns Hopkins University Press (3rd Edition), Baltimore

5 Parameters

1: UPLO — CHARACTER*1 *Input*

On entry: indicates how A has been factorized as follows:

if $\text{UPLO} = \text{'U}'$, then $A = PUDU^T P^T$, where U is upper triangular;
 if $\text{UPLO} = \text{'L}'$, then $A = PLDL^T P^T$, where L is lower triangular.

Constraint: $\text{UPLO} = \text{'U'}$ or 'L' .

2: N — INTEGER *Input*

On entry: n , the order of the matrix A .

Constraint: $N \geq 0$.

3: NRHS — INTEGER *Input*

On entry: r , the number of right-hand sides.

Constraint: $\text{NRHS} \geq 0$.

4:	A(LDA,*) — complex array	<i>Input</i>
Note: the second dimension of the array A must be at least $\max(1,N)$.		
<i>On entry:</i> details of the factorization of A , as returned by F07NRF (CSYTRF/ZSYTRF).		
5:	LDA — INTEGER	<i>Input</i>
<i>On entry:</i> the first dimension of the array A as declared in the (sub)program from which F07NSF (CSYTRS/ZSYTRS) is called.		
<i>Constraint:</i> $LDA \geq \max(1,N)$.		
6:	IPIV(*) — INTEGER array	<i>Input</i>
Note: the dimension of the array IPIV must be at least $\max(1,N)$.		
<i>On entry:</i> details of the interchanges and the block structure of D , as returned by F07NRF (CSYTRF/ZSYTRF).		
7:	B(LDB,*) — complex array	<i>Input/Output</i>
Note: the second dimension of the array B must be at least $\max(1,NRHS)$.		
<i>On entry:</i> the n by r right-hand side matrix B .		
<i>On exit:</i> the n by r solution matrix X .		
8:	LDB — INTEGER	<i>Input</i>
<i>On entry:</i> the first dimension of the array B as declared in the (sub)program from which F07NSF (CSYTRS/ZSYTRS) is called.		
<i>Constraint:</i> $LDB \geq \max(1,N)$.		
9:	INFO — INTEGER	<i>Output</i>
<i>On exit:</i> INFO = 0 unless the routine detects an error (see Section 6).		

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$\begin{aligned}|E| &\leq c(n)\epsilon P|U||D||U^T|P^T && \text{if } \text{UPLO} = \text{'U'}, \\ |E| &\leq c(n)\epsilon P|L||D||L^T|P^T && \text{if } \text{UPLO} = \text{'L'},\end{aligned}$$

$c(n)$ is a modest linear function of n , and ϵ is the **machine precision**.

If \hat{x} is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\text{cond}(A, x)\epsilon$$

where $\text{cond}(A, x) = \|A^{-1}\|A\|x\|_\infty/\|x\|_\infty \leq \text{cond}(A) = \|A^{-1}\|A\|_\infty \leq \kappa_\infty(A)$. Note that $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling F07NVF (CSYRFS/ZSYRFS), and an estimate for $\kappa_\infty(A)$ ($= \kappa_1(A)$) can be obtained by calling F07NUF (CSYCON/ZSYCON).

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This routine may be followed by a call to F07NVF (CSYRFS/ZSYRFS) to refine the solution and return an error estimate.

The real analogue of this routine is F07MEF (SSYTRS/DSYTRS).

9 Example

To solve the system of equations $AX = B$, where

$$A = \begin{pmatrix} -0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\ 5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\ -7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\ 3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -55.64 + 41.22i & -19.09 - 35.97i \\ -48.18 + 66.00i & -12.08 - 27.02i \\ -0.49 - 1.47i & 6.95 + 20.49i \\ -6.43 + 19.24i & -4.59 - 35.53i \end{pmatrix}.$$

Here A is symmetric and must first be factorized by F07NRF (CSYTRF/ZSYTRF).

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```

*      F07NSF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*
*      ... Parameters ...
INTEGER           NIN, NOUT
PARAMETER        (NIN=5,NOUT=6)
INTEGER           NMAX, LDA, LWORK, NRHMAX, LDB
PARAMETER        (NMAX=8,LDA=NMAX,LWORK=64*NMAX,NRHMAX=NMAX,
+                  LDB=NMAX)
*
*      ... Local Scalars ...
INTEGER           I, IFAIL, INFO, J, N, NRHS
CHARACTER         UPTO
*
*      ... Local Arrays ...
complex          A(LDA,NMAX), B(LDB,NRHMAX), WORK(LWORK)
INTEGER           IPIV(NMAX)
CHARACTER         CLABS(1), RLABS(1)
*
*      ... External Subroutines ...
EXTERNAL          csytrf, csytrs, X04DBF
*
*      ... Executable Statements ...
WRITE (NOUT,*) 'F07NSF Example Program Results'
*
*      Skip heading in data file
READ (NIN,*) N, NRHS
READ (NIN,*) N, NRHS
IF (N.LE.NMAX .AND. NRHS.LE.NRHMAX) THEN
*
*      Read A and B from data file
*
READ (NIN,*) UPTO
IF (UPTO.EQ.'U') THEN
    READ (NIN,*) ((A(I,J),J=I,N),I=1,N)

```

```

        ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((A(I,J),J=1,I),I=1,N)
        END IF
        READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
*      Factorize A
*
        CALL csytrf(UPLO,N,A,LDA,IPIV,WORK,LWORK,INFO)
*
        WRITE (NOUT,*)
        IF (INFO.EQ.0) THEN
*
*      Compute solution
*
        CALL csytrs(UPLO,N,NRHS,A,LDA,IPIV,B,LDB,INFO)
*
*      Print solution
*
        IFAIL = 0
        CALL X04DBF('General', ' ', N, NRHS, B, LDB, 'Bracketed', 'F7.4',
+                  'Solution(s)', 'Integer', RLABS, 'Integer', CLABS,
+                  80, 0, IFAIL)
        ELSE
            WRITE (NOUT,*) 'The factor D is singular'
        END IF
    END IF
    STOP
*
    END

```

9.2 Program Data

```

F07NSF Example Program Data
 4 2                                     :Values of N and NRHS
'L'                                       :Value of UPLO
(-0.39,-0.71)
( 5.14,-0.64) ( 8.86, 1.81)
(-7.86,-2.96) (-3.52, 0.58) (-2.83,-0.03)
( 3.80, 0.92) ( 5.32,-1.59) (-1.54,-2.86) (-0.56, 0.12)  :End of matrix A
(-55.64, 41.22) (-19.09,-35.97)
(-48.18, 66.00) (-12.08,-27.02)
( -0.49, -1.47) ( 6.95, 20.49)
( -6.43, 19.24) ( -4.59,-35.53)                               :End of matrix B

```

9.3 Program Results

F07NSF Example Program Results

Solution(s)	1	2
1	(1.0000,-1.0000)	(-2.0000,-1.0000)
2	(-2.0000, 5.0000)	(1.0000,-3.0000)
3	(3.0000,-2.0000)	(3.0000, 2.0000)
4	(-4.0000, 3.0000)	(-1.0000, 1.0000)