

## G13DSF – NAG Fortran Library Routine Document

**Note.** Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

### 1 Purpose

G13DSF is a diagnostic checking routine suitable for use after fitting a vector ARMA model to a multivariate time series using G13DCF. The residual cross-correlation matrices are returned along with an estimate of their asymptotic standard errors and correlations. Also, G13DSF calculates the modified Li–McLeod portmanteau statistic and its significance level for testing model adequacy.

### 2 Specification

```

SUBROUTINE G13DSF(K, N, V, IK, IP, IQ, M, PAR, PARHLD, QQ, ISHOW,
1          RO, R, RCM, IRCM, CHI, IDF, SIGLEV, IW, LIW,
2          WORK, LWORK, IFAIL)
  INTEGER  K, N, IK, IP, IQ, M, ISHOW, IRCM, IDF, IW(LIW),
1          LIW, LWORK, IFAIL
  real    V(IK,N), PAR((IP+IQ)*K*K), QQ(IK,K), RO(IK,K),
1          R(IK,IK,M), RCM(IRCM,M*K*K), CHI, SIGLEV,
2          WORK(LWORK)
  LOGICAL  PARHLD((IP+IQ)*K*K)

```

### 3 Description

Let  $W_t = (w_{1t}, w_{2t}, \dots, w_{kt})^T$ , for  $t = 1, 2, \dots, n$  denote a vector of  $k$  time series which is assumed to follow a multivariate ARMA model of the form:

$$W_t - \mu = \phi_1(W_{t-1} - \mu) + \phi_2(W_{t-2} - \mu) + \dots + \phi_p(W_{t-p} - \mu) + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \dots - \theta_q\epsilon_{t-q} \quad (1)$$

where  $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \dots, \epsilon_{kt})^T$ , for  $t = 1, 2, \dots, n$  is a vector of  $k$  residual series assumed to be normally distributed with zero mean and positive-definite covariance matrix  $\Sigma$ . The components of  $\epsilon_t$  are assumed to be uncorrelated at non-simultaneous lags. The  $\phi_i$ 's and  $\theta_j$ 's are  $k$  by  $k$  matrices of parameters.  $\{\phi_i\}$ , for  $i = 1, 2, \dots, p$ , are called the autoregressive (AR) parameter matrices, and  $\{\theta_i\}$ , for  $i = 1, 2, \dots, q$ , the moving average (MA) parameter matrices. The parameters in the model are thus the  $p$  ( $k$  by  $k$ )  $\phi$ -matrices, the  $q$  ( $k$  by  $k$ )  $\theta$ -matrices, the mean vector  $\mu$  and the residual error covariance matrix  $\Sigma$ . Let

$$A(\phi) = \begin{bmatrix} \phi_1 & I & 0 & \cdot & \cdot & \cdot & 0 \\ \phi_2 & 0 & I & 0 & \cdot & \cdot & 0 \\ \cdot & & & \cdot & & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & & \cdot & \\ \phi_{p-1} & 0 & \cdot & \cdot & \cdot & 0 & I \\ \phi_p & 0 & \cdot & \cdot & \cdot & 0 & 0 \end{bmatrix}_{pk \times pk} \quad \text{and} \quad B(\theta) = \begin{bmatrix} \theta_1 & I & 0 & \cdot & \cdot & \cdot & 0 \\ \theta_2 & 0 & I & 0 & \cdot & \cdot & 0 \\ \cdot & & & \cdot & & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & & \cdot & \\ \theta_{q-1} & 0 & \cdot & \cdot & \cdot & \cdot & I \\ \theta_q & 0 & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix}_{qk \times qk}$$

where  $I$  denotes the  $k$  by  $k$  identity matrix.

The model (1) is said to be stationary if the eigenvalues of  $A(\phi)$  lie inside the unit circle, and invertible if the eigenvalues of  $B(\theta)$  lie inside the unit circle. The ARMA model is assumed to be both stationary and invertible. Note that some of the elements of the  $\phi$ - and/or  $\theta$ -matrices may have been fixed at pre-specified values (for example by calling G13DCF).

The estimated residual cross-correlation matrix at lag  $l$  is defined to the  $k$  by  $k$  matrix  $\hat{R}_l$  whose  $(i, j)$ th

element is computed as

$$\hat{r}_{ij}(l) = \frac{\sum_{t=l+1}^n (\hat{\epsilon}_{it-l} - \bar{\epsilon}_i)(\hat{\epsilon}_{jt} - \bar{\epsilon}_j)}{\sqrt{\sum_{t=1}^n (\hat{\epsilon}_{it} - \bar{\epsilon}_i)^2 \sum_{t=1}^n (\hat{\epsilon}_{jt} - \bar{\epsilon}_j)^2}}, \quad l = 0, 1, \dots; i, j = 1, 2, \dots, k$$

where  $\hat{\epsilon}_{it}$ , denotes an estimate of the  $t$ th residual for the  $i$ th series,  $\epsilon_{it}$ , and  $\bar{\epsilon}_i = \sum_{t=1}^n \hat{\epsilon}_{it}/n$ . (Note that  $\hat{R}_l$  is an estimate of  $E(\epsilon_{t-l}\epsilon_t^T)$ , where  $E$  is the expected value.)

A modified portmanteau statistic,  $Q_{(m)}^*$ , is calculated from the formula (see Li and McLeod [1]):

$$Q_{(m)}^* = \frac{k^2 m(m+1)}{2n} + n \sum_{l=1}^m \hat{r}(l)^T (\hat{R}_0^{-1} \otimes \hat{R}_0^{-1}) \hat{r}(l)$$

where  $\otimes$  denotes kronecker product,  $\hat{R}_0$  is the estimated residual cross-correlation matrix at lag zero and  $\hat{r}(l) = \text{vec}(\hat{R}_l^T)$  where  $\text{vec}$  of a  $k$  by  $k$  matrix is a vector with the  $(i, j)$ th element in position  $(i-1)k + j$ .  $m$  denotes the number of residual cross-correlation matrices computed. (Advice on the choice of  $m$  is given in Section 8.2). Let  $l_C$  denote the total number of ‘free’ parameters in the ARMA model excluding the mean,  $\mu$ , and the residual error covariance matrix  $\Sigma$ . Then, under the hypothesis of model adequacy,  $Q_{(m)}^*$ , has an asymptotic  $\chi^2$  distribution on  $mk^2 - l_C$  degrees of freedom.

Let  $\hat{\underline{r}} = (\text{vec}(R_1^T), \text{vec}(R_2^T), \dots, \text{vec}(R_m^T))$  then the covariance matrix of  $\hat{\underline{r}}$  is given by

$$\text{Var}(\hat{\underline{r}}) = [Y - X(X^T G G^T X)^{-1} X^T]/n$$

where  $Y = I_m \otimes (\Delta \otimes \Delta)$  and  $G = I_m (G G^T)$ .  $\Delta$  is the dispersion matrix  $\Sigma$  in correlation form and  $G$  a non-singular  $k$  by  $k$  matrix such that  $G G^T = \Delta^{-1}$  and  $G \Delta G^T = I_k$ . The construction of the matrix  $X$  is discussed in Li and McLeod [1]. (Note that the mean,  $\mu$ , plays no part in calculating  $\text{Var}(\hat{\underline{r}})$  and therefore is not required as input to G13DSF.)

## 4 References

- [1] Li W K and McLeod A I (1981) Distribution of the residual autocorrelations in multivariate ARMA time series models *J. Roy. Statist. Soc. Ser. B* **43** 231–239

## 5 Parameters

The output quantities K, N, V, IK, IP, IQ, PAR, PARHLD and QQ from G13DCF are suitable for input to G13DSF.

- 1:** K — INTEGER *Input*  
*On entry:* the number of residual time series,  $k$ .  
*Constraint:*  $K \geq 1$ .
- 2:** N — INTEGER *Input*  
*On entry:* the number of observations in each residual series,  $n$ .  
*Constraint:*  $N \geq 3$ .
- 3:** V(IK,N) — *real* array *Input*  
*On entry:* an estimate of the  $i$ th component of  $\epsilon_t$ , for  $i = 1, 2, \dots, k; t = 1, 2, \dots, n$ .  
*Constraints:*  
 no two rows of  $V$  may be identical.  
 In each row there must be at least two distinct elements.

- 4:** IK — INTEGER *Input*  
*On entry:* the first dimension of the arrays V, QQ and R0 as declared in the (sub)program from which G13DSF is called.  
*Constraint:*  $IK \geq K$ .
- 5:** IP — INTEGER *Input*  
*On entry:* the number of AR parameter matrices,  $p$ .  
*Constraint:*  $IP \geq 0$ .
- 6:** IQ — INTEGER *Input*  
*On entry:* the number of MA parameter matrices,  $q$ .  
*Constraint:*  $IQ \geq 0$ .
- Note.**  $IP = IQ = 0$  is **not permitted**.
- 7:** M — INTEGER *Input*  
*On entry:* the value of  $m$ , the number of residual cross-correlation matrices to be computed. See Section 8.2 for advice on the choice of  $M$ .  
*Constraint:*  $IP + IQ < M < N$ .
- 8:** PAR((IP+IQ)\*K\*K) — *real* array *Input*  
*On entry:* the parameter estimates read in row by row in the order  $\phi_1, \phi_2, \dots, \phi_p, \theta_1, \theta_2, \dots, \theta_q$ . Thus, if  $IP > 0$  then  $PAR((l-1) \times k \times k + (i-1) \times k + j)$  must be set equal to an estimate of the  $(i, j)$ th element of  $\phi_l$  for  $l = 1, 2, \dots, p$ ;  $i, j = 1, 2, \dots, k$ . If  $IQ \geq 0$  then  $PAR(p \times k \times k + (l-1) \times k \times k + (i-1) \times k + j)$  must be set equal to an estimate of the  $(i, j)$ th element of  $\theta_l$  for  $l = 1, 2, \dots, q$ ;  $i, j = 1, 2, \dots, k$ .
- The first  $p \times k \times k$  elements of PAR must satisfy the stationarity condition and the next  $q \times k \times k$  elements of PAR must satisfy the invertibility condition.
- 9:** PARHLD((IP+IQ)\*K\*K) — LOGICAL array *Input*  
*On entry:* PARHLD( $i$ ) must be set to .TRUE. if PAR( $i$ ) has been held constant at a pre-specified value and .FALSE. if PAR( $i$ ) is a free parameter, for  $i = 1, 2, \dots, (p+q) \times k \times k$ .
- 10:** QQ(IK,K) — *real* array *Input/Output*  
*On entry:* an efficient estimate of the  $(i, j)$ th element of  $\Sigma$ . The lower triangle only is needed.  
*Constraint:* QQ must be positive-definite.  
*On exit:* if IFAIL  $\neq 1$ , then the upper triangle is set equal to the lower triangle.
- 11:** ISHOW — INTEGER *Input*  
*On entry:* ISHOW must be non-zero if the residual cross-correlation matrices  $\{\hat{r}_{ij}(l)\}$  and their standard errors  $\{se(\hat{r}_{ij}(l))\}$ , the modified portmanteau statistic with its significance and a summary table are to be printed and zero otherwise. The summary table indicates which elements of the residual correlation matrices are significant at the 5% level in either a positive or negative direction; i.e., if  $\hat{r}_{ij}(l) > 1.96 \times se(\hat{r}_{ij}(l))$  then a '+' is printed, if  $\hat{r}_{ij}(l) < -1.96 \times se(\hat{r}_{ij}(l))$  then a '-' is printed, otherwise a '.' is printed. The summary table is only printed if  $k \leq 6$  on entry.
- The residual cross-correlation matrices, their standard errors and the modified portmanteau statistic with its significance are available also as output variables in R, RCM, CHI, IDF and SIGLEV.
- 12:** R0(IK,K) — *real* array *Output*  
*On exit:* if  $i \neq j$ , then  $R0(i, j)$  contains an estimate of the  $(i, j)$ th element of the residual cross-correlation matrix at lag zero,  $\hat{R}_0$ . When  $i = j$ ,  $R0(i, j)$  contains the standard deviation of the  $i$ th residual series. If IFAIL = 3 on exit then the first  $K$  rows and columns of R0 are set to zero.

- 13:** R(IK,IK,M) — *real* array Output  
*On exit:* an estimate of the  $(i, j)$ th element of the residual cross-correlation matrix at lag  $l$ , for  $l = 1, 2, \dots, m$ ;  $i, j = 1, 2, \dots, k$ . If IFAIL = 3 on exit then all elements of R are set to zero.
- 14:** RCM(IRCM,M\*K\*K) — *real* array Output  
*On exit:* the estimated standard errors and correlations of the elements in the array R. The correlation between  $R(i, j, l)$  and  $R(i_2, j_2, l_2)$  is returned as  $RCM(s, t)$  where  $s = (l - 1) \times k \times k + (j - 1) \times k + i$  and  $t = (l_2 - 1) \times k \times k + (j_2 - 1) \times k + i_2$  except that if  $s = t$ , then  $RCM(s, t)$  contains the standard error of  $R(i, j, l)$ . If on exit,  $IFAIL \geq 5$ , then all off-diagonal elements of RCM are set to zero and all diagonal elements are set to  $1/\sqrt{n}$ .
- 15:** IRCM — INTEGER Input  
*On entry:* the first dimension of the array RCM as declared in the (sub)program from which G13DSF is called.  
*Constraint:*  $IRCM \geq M \times K \times K$ .
- 16:** CHI — *real* Output  
*On exit:* the value of the modified portmanteau statistic,  $Q_{(m)}^*$ . If IFAIL = 3 on exit then CHI is returned as zero.
- 17:** IDF — INTEGER Output  
*On exit:* the number of degrees of freedom of CHI.
- 18:** SIGLEV — *real* Output  
*On exit:* the significance level of CHI based on IDF degrees of freedom. If IFAIL = 3 on exit then SIGLEV is returned as one.
- 19:** IW(LIW) — INTEGER array Workspace
- 20:** LIW — INTEGER Input  
*On entry:* the dimension of the array IW as declared in the (sub)program from which G13DSF is called.  
*Constraint:*  $LIW \geq K \times \max(IP, IQ)$ .
- 21:** WORK(LWORK) — *real* array Workspace
- 22:** LWORK — INTEGER Input  
*On entry:* the dimension of the array WORK as declared in the (sub)program from which G13DSF is called.  
*Constraint:*  
if  $NPAR = (p + q)k^2$  then,  
 $LWORK \geq k(n + IK + 2) + mk^2(NPAR + mk^2 + 1) + 3k^2 + (NPAR + 1)NPAR$ .
- 23:** IFAIL — INTEGER Input/Output  
*On entry:* IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.  
*On exit:* IFAIL = 0 unless the routine detects an error or gives a warning (see Section 6).

**For this routine**, because the values of output parameters may be useful even if IFAIL  $\neq$  0 on exit, users are recommended to set IFAIL to -1 before entry. **It is then essential to test the value of IFAIL on exit.**

## 6 Error Indicators and Warnings

If on entry `IFAIL = 0` or `-1`, explanatory error messages are output on the current error message unit (as defined by `X04AAF`).

Errors or warnings specified by the routine:

`IFAIL = 1`

- On entry,  $K < 1$ ,
- or  $IK < K$ ,
- or  $IP < 0$ ,
- or  $IQ < 0$ ,
- or  $IP = IQ = 0$ ,
- or  $M \leq IP + IQ$ ,
- or  $M \geq N$ ,
- or  $IRCM < M \times K \times K$ ,
- or `LIW` is too small,
- or `LWORK` is too small.

`IFAIL = 2`

On entry, either `QQ` is not positive-definite or the autoregressive parameter matrices are extremely close to or outside the stationarity region, or the moving average parameter matrices are extremely close to or outside the invertibility region. To proceed, the user must supply different parameter estimates in the arrays `PAR` and `QQ`.

`IFAIL = 3`

On entry, at least one of the  $k$  residual series is such that all its elements are practically identical giving zero (or near zero) variance or at least two of the residual series are identical. In this case `CHI` is set to zero, `SIGLEV` to one and all the elements of `R0` and `R` set to zero.

`IFAIL = 4`

This is an unlikely exit brought about by an excessive number of iterations being needed to evaluate the zeros of the determinantal polynomials  $\det(A(\phi))$  and  $\det(B(\theta))$ . All output parameters are undefined.

`IFAIL = 5`

On entry, either the eigenvalues and eigenvectors of  $\Delta$  (the matrix `QQ` in correlation form) could not be computed or the determinantal polynomials  $\det(A(\phi))$  and  $\det(B(\theta))$  have a factor in common. To proceed, the user must either supply different parameter estimates in the array `QQ` or delete this common factor from the model. In this case, the off-diagonal elements of `RCM` are returned as zero and the diagonal elements set to  $1/\sqrt{n}$ . All other output quantities will be correct.

`IFAIL = 6`

This is an unlikely exit. At least one of the diagonal elements of `RCM` was found to be either negative or zero. In this case all off-diagonal elements of `RCM` are returned as zero and all diagonal elements of `RCM` set to  $1/\sqrt{n}$ .

## 7 Accuracy

The computations are believed to be stable.

## 8 Further Comments

### 8.1 Timing

The time taken by the routine depends upon the number of residual cross-correlation matrices to be computed,  $m$ , and the number of time series,  $k$ .

## 8.2 Choice of $m$

The number of residual cross-correlation matrices to be computed,  $m$  should be chosen to ensure that when the ARMA model (1) is written as either an infinite order autoregressive process:

$$\text{i.e. } W_t - \mu = \sum_{j=1}^{\infty} \pi_j (W_{t-j} - \mu) + \epsilon_t$$

or as an infinite order moving average process:

$$\text{i.e. } W_t - \mu = \sum_{j=1}^{\infty} \psi_j \epsilon_{t-j} + \epsilon_t$$

then the two sequences of  $k$  by  $k$  matrices  $\{\pi_1, \pi_2, \dots\}$  and  $\{\psi_1, \psi_2, \dots\}$  are such that  $\pi_j$  and  $\psi_j$  are approximately zero for  $j > m$ . An over-estimate of  $m$  is therefore preferable to an under-estimate of  $m$ . In many instances the choice  $m = 10$  will suffice. In practice, to be on the safe side, the user should try setting  $m = 20$ .

## 8.3 Checking a ‘White Noise’ Model

If the user has fitted the ‘white noise’ model

$$W_t - \mu = \epsilon_t$$

then G13DSF should be entered with  $p = 1$ ,  $q = 0$ ; and the first  $k^2$  elements of PAR and PARHLD set to zero and .TRUE. respectively.

## 8.4 Approximate Standard Errors

When IFAIL is returned as 5 or 6 all the standard errors in RCM are set to  $1/\sqrt{n}$ . This is the asymptotic standard error of  $\hat{r}_{ij}(l)$  when all the autoregressive and moving average parameters are assumed to be known rather than estimated.

## 8.5 Alternative Tests

$\hat{R}_0$  is useful in testing for instantaneous causality. If the user wishes to carry out a likelihood ratio test then the covariance matrix at lag zero ( $\hat{C}_0$ ) can be used. It can be recovered from  $\hat{R}_0$  by setting:

$$\begin{aligned} \hat{C}_0(i, j) &= \hat{R}_0(i, j) \times \hat{R}_0(i, i) \times \hat{R}_0(j, j), & \text{for } i \neq j \\ &= \hat{R}_0(i, j) \times \hat{R}_0(i, j), & \text{for } i = j \end{aligned}$$

## 9 Example

A program to fit a bivariate AR(1) model to two series each of length 48.  $\mu$  has been estimated but  $\phi_1(2, 1)$  has been constrained to be zero. Ten residual cross-correlation matrices are to be computed.

### 9.1 Program Text

**Note.** The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users’ Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      G13DSF Example Program Text
*      Mark 15 Revised.  NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER       (NIN=5,NOUT=6)
      INTEGER          KMAX, IK, IPMAX, IQMAX, NMAX, NPARMX, LWORK, LIW,
+                   MMAX, IRCM, ICM
```

```

PARAMETER      (KMAX=3,IK=KMAX,IPMAX=3,IQMAX=3,NMAX=100,
+              NPARMX=(IPMAX+IQMAX)*KMAX*KMAX+KMAX,LWORK=2000,
+              LIW=100,MMAX=20,IRCM=MMAX*KMAX*KMAX,ICM=NPARMX)
*
.. Local Scalars ..
  real          CGETOL, CHI, RLOGL, SIGLEV
INTEGER        I, IDF, IFAIL, IP, IPRINT, IQ, ISHOW, J, K, M,
+             MAXCAL, N, NITER, NPAR
LOGICAL        EXACT, MEAN
*
.. Local Arrays ..
  real          CM(ICM,NPARMX), G(NPARMX), PAR(NPARMX),
+             QQ(IK,KMAX), R(IK,IK,MMAX), RO(IK,KMAX),
+             RCM(IRCM,MMAX*KMAX*KMAX), V(IK,NMAX), W(IK,NMAX),
+             WORK(LWORK)
INTEGER        IW(LIW)
LOGICAL        PARHLD(NPARMX)
*
.. External Subroutines ..
EXTERNAL       G13DCF, G13DSF, X04ABF
*
.. Executable Statements ..
WRITE (NOUT,*) 'G13DSF Example Program Results'
*
Skip heading in data file
READ (NIN,*)
READ (NIN,*) K, N
*
CALL X04ABF(1,NOUT)
*
IF (K.GT.0 .AND. K.LE.KMAX .AND. N.GE.3 .AND. N.LE.NMAX) THEN
  DO 20 I = 1, K
    READ (NIN,*) (W(I,J),J=1,N)
20  CONTINUE
  READ (NIN,*) IP, IQ, MEAN, M
  IF (IP.GE.0 .AND. IP.LE.IPMAX .AND. IQ.GE.0 .AND. IQ.LE.IQMAX)
+   THEN
    NPAR = (IP+IQ)*K*K
    IF (MEAN) NPAR = NPAR + K
    IF (NPAR.LE.NPARMX) THEN
      DO 40 I = 1, NPAR
        PAR(I) = 0.0e0
        PARHLD(I) = .FALSE.
40    CONTINUE
      DO 80 J = 1, K
        DO 60 I = J, K
          QQ(I,J) = 0.0e0
60    CONTINUE
80    CONTINUE
      PARHLD(3) = .TRUE.
      EXACT = .TRUE.
*
      ** Set IPRINT $>$ 0 to obtain intermediate output **
      IPRINT = -1
      CGETOL = 0.0001e0
      MAXCAL = 40*NPAR*(NPAR+5)
      ISHOW = 2
      IFAIL = 1
*
      CALL G13DCF(K,N,IP,IQ,MEAN,PAR,NPAR,QQ,IK,W,PARHLD,EXACT,
+             IPRINT,CGETOL,MAXCAL,ISHOW,NITER,RLOGL,V,G,
+             CM,ICM,WORK,LWORK,IW,LIW,IFAIL)
*
      WRITE (NOUT,*)

```

```

      IF (IFAIL.NE.0) THEN
        WRITE (NOUT,99999) 'G13DCF fails. IFAIL =', IFAIL
        WRITE (NOUT,*)
      END IF
      IF ((IFAIL.EQ.0 .OR. IFAIL.GE.4) .AND. M.LE.MMAX) THEN
        WRITE (NOUT,*) 'Output from G13DSF'
        WRITE (NOUT,*)
        ISHOW = 1
        IFAIL = -1
*
        CALL G13DSF(K,N,V,IK,IP,IQ,M,PAR,PARHLD,QQ,ISHOW,RO,R,
+             RCM,IRCM,CHI,IDF,SIGLEV,IW,LIW,WORK,LWORK,
+             IFAIL)
*
        IF (IFAIL.NE.0) WRITE (NOUT,99999)
+             'G13DSF fails. IFAIL =', IFAIL
      END IF
    END IF
  END IF
END IF
STOP
*
99999 FORMAT (1X,A,I3)
      END

```

## 9.2 Program Data

### G13DSF Example Program Data

```

2 48 : K, no. of time series, N, no. of obs in each time series
-1.490 -1.620 5.200 6.230 6.210 5.860
4.090 3.180 2.620 1.490 1.170 0.850
-0.350 0.240 2.440 2.580 2.040 0.400
2.260 3.340 5.090 5.000 4.780 4.110
3.450 1.650 1.290 4.090 6.320 7.500
3.890 1.580 5.210 5.250 4.930 7.380
5.870 5.810 9.680 9.070 7.290 7.840
7.550 7.320 7.970 7.760 7.000 8.350
7.340 6.350 6.960 8.540 6.620 4.970
4.550 4.810 4.750 4.760 10.880 10.010
11.620 10.360 6.400 6.240 7.930 4.040
3.730 5.600 5.350 6.810 8.270 7.680
6.650 6.080 10.250 9.140 17.750 13.300
9.630 6.800 4.080 5.060 4.940 6.650
7.940 10.760 11.890 5.850 9.010 7.500
10.020 10.380 8.150 8.370 10.730 12.140 : End of time series
1 0 T 10 : IP, IQ, MEAN and M

```

### 9.3 Program Results

G13DSF Example Program Results

VALUE OF IFAIL PARAMETER ON EXIT FROM G13DCF = 0

VALUE OF LOG LIKELIHOOD FUNCTION ON EXIT = -0.20280E+03

MAXIMUM LIKELIHOOD ESTIMATES OF AR PARAMETER MATRICES

-----  
 PHI(1) ROW-WISE : 0.802 0.065  
 (0.091) (0.102)

0.000 0.575  
 (0.000) (0.121)

MAXIMUM LIKELIHOOD ESTIMATE OF PROCESS MEAN

-----  
 4.271 7.825  
 (1.219) (0.776)

MAXIMUM LIKELIHOOD ESTIMATE OF SIGMA MATRIX

-----  
 2.964

0.637 5.380

RESIDUAL SERIES NUMBER 1

-----  
 T 1 2 3 4 5 6 7 8  
 V(T) -3.33 -1.24 5.75 1.27 0.32 0.11 -1.27 -0.73  
 T 9 10 11 12 13 14 15 16  
 V(T) -0.58 -1.26 -0.67 -1.13 -2.02 -0.57 1.24 -0.13  
 T 17 18 19 20 21 22 23 24  
 V(T) -0.77 -2.09 1.34 0.95 1.71 0.23 -0.01 -0.60  
 T 25 26 27 28 29 30 31 32  
 V(T) -0.68 -1.89 -0.77 2.05 2.11 0.94 -3.32 -2.50  
 T 33 34 35 36 37 38 39 40  
 V(T) 3.16 0.47 0.05 2.77 -0.82 0.25 3.99 0.20  
 T 41 42 43 44 45 46 47 48  
 V(T) -0.70 1.07 0.44 0.28 1.09 0.50 -0.10 1.70

## RESIDUAL SERIES NUMBER 2

```

-----
T      1      2      3      4      5      6      7      8
V(T) -0.19 -1.20 -0.02  1.21 -1.62 -2.16 -1.63 -1.13

T      9     10     11     12     13     14     15     16
V(T) -1.34 -1.30  4.82  0.43  2.54  0.35 -2.88 -0.77

T     17     18     19     20     21     22     23     24
V(T)  1.02 -3.85 -1.92  0.13 -1.20  0.41  1.03 -0.40

T     25     26     27     28     29     30     31     32
V(T) -1.09 -1.07  3.43 -0.08  9.17 -0.23 -1.34 -2.06

T     33     34     35     36     37     38     39     40
V(T) -3.16 -0.61 -1.30  0.48  0.79  2.87  2.38 -4.31

T     41     42     43     44     45     46     47     48
V(T)  2.32 -1.01  2.38  1.29 -1.14  0.36  2.59  2.64

```

Output from G13DSF

## RESIDUAL CROSS-CORRELATION MATRICES

```

-----
LAG    1          :   0.130  0.112
                   (0.119) (0.143)
                   0.094  0.043
                   (0.069) (0.102)

LAG    2          :  -0.312  0.021
                   (0.128) (0.144)
                   -0.162  0.098
                   (0.125) (0.132)

LAG    3          :   0.004 -0.176
                   (0.134) (0.144)
                   -0.168 -0.091
                   (0.139) (0.140)

LAG    4          :  -0.090 -0.120
                   (0.137) (0.144)
                   0.099 -0.232
                   (0.142) (0.143)

LAG    5          :   0.041  0.093
                   (0.140) (0.144)
                   -0.009 -0.089
                   (0.144) (0.144)

LAG    6          :   0.234 -0.008
                   (0.141) (0.144)
                   0.069 -0.103
                   (0.144) (0.144)

```

```

LAG      7      :   -0.076   0.007
                (0.142) (0.144)
                0.168   0.000
                (0.144) (0.144)

LAG      8      :   -0.074   0.559
                (0.143) (0.144)
                0.008  -0.101
                (0.144) (0.144)

LAG      9      :    0.091   0.193
                (0.144) (0.144)
                0.055   0.170
                (0.144) (0.144)

LAG     10      :   -0.060   0.061
                (0.144) (0.144)
                0.191   0.089
                (0.144) (0.144)

```

## SUMMARY TABLE

-----

LAGS 1 - 10

```

*****
*           *           *
* .-..... * .....+. *
*           *           *
*****
*           *           *
* ..... * ..... *
*           *           *
*****

```

```

LI-MCLEOD PORTMANTEAU STATISTIC = 49.234
SIGNIFICANCE LEVEL = 0.086
(BASED ON 37 DEGREES OF FREEDOM)

```

```

VALUE OF IFAIL PARAMETER ON EXIT FROM G13DSF = 0

```